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1989 J. Phys.: Condens. Matter 1 5391

(http://iopscience.iop.org/0953-8984/1/32/008)

J. Phys.: Condens. Matter **1** (1989) 5391–5401. Printed in the UK

Propagating electromagnetic modes in thin semiconductor films

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Received 3 October 1988, in final form 3 February 1989

Abstract. The dispersion relations of plasmon–polariton modes in quantised thin but finite semiconductor films are derived and their characteristics features are discussed. It is shown that the lowest two-dimensional unretarded plasma frequency increases strongly with decreasing film thickness in the presence of image charge forces. The dispersion relations are calculated and illustrated for GaAs and PbTe films. Calculations of the non-radiative dispersion spectra indicate that both the transverse magnetic (p-polarised) and transverse electric (s-polarised) plasmon–polariton dispersion spectra possess some gaps. The value of the gap depends on the thickness of the film and carrier surface concentration. It is found that, in thin but finite films, guided and surface localised polariton transverse magnetic modes coincide. For transverse electric modes the coupling between external electromagnetic fields and carriers is weak and almost insignificant.

1. Introduction

Current interest in quasi-two-dimensionally (2D) confined charge carriers has been enhanced by the recent advances made in the experimental creation of such systems in quantum wells, heterojunctions and superlattices. A great deal of interest has been attached to the theoretical study of collective oscillations of electrons in these systems. The dispersion relations for collective plasmon modes have been investigated for both low-frequency intra-sub-band (Stern 1967, Fetter 1973, Nakayama 1974) and high-frequency inter-sub-band excitations (Chen *et al* 1976, Allen *et al* 1976) in semiconductor quantum wells. In particular, it has been shown that in the non-retarded limit the low-frequency 2D plasmon associated with the ground sub-band has dispersion

$$\omega \sim q^{1/2}/\varepsilon, \tag{1.1}$$

where q is the in-plane wavevector and ε the permittivity. The high-frequency inter-band excitation branches are shifted from the sub-band separation energies owing to polarisation.

Dahl and Sham (1977) and Eguiluz and Maradudin (1978) gave a more complete analysis of a quasi-2D gas in thin but finite semiconductor inversion layers by means of the random phase approximation (RPA) including the effect of retardation. They constructed the non-local frequency-dependent dielectric tensor in the RPA and used the Maxwell equations to study the electromagnetic surface properties of the quantum wells.

Beck and Kumar (1976) and Dahl and Sham (1977) have included the effect of the image charges (EICH) and noted that in the important long-wavelength limit the EICH appears in the dispersion relations for the low-frequency region only. Even there this effect is comparatively weak and amounts to replacing ε by $(\varepsilon + \varepsilon_{ins})/2$, where ε_{ins} is the oxide dielectric permittivity.

Very recently it has been shown (Aharonian *et al* 1988) that the EICH on the collective plasmon spectra in thin semiconductor films does lead to radical changes if the dielectric permittivity ε of the semiconductor is much larger than that of the substrate.

Aharonian *et al* showed that in the long-wave limit ($qd \ll 1$) the lowest non-retarded 2D plasmon frequency is

$$\omega \sim (1/\varepsilon)\{q/[qd + (\varepsilon_1 + \varepsilon_2)/\varepsilon]\}^{1/2} \tag{1.2}$$

which increases strongly with decreasing film thickness d. In (2), $\varepsilon_{1,2}$ are the permittivities of the bounding media. In particular, (2) has the property that in the wavevector region

$$1 \gg qd \gg (\varepsilon_1 + \varepsilon_2)/\varepsilon$$

ω is independent of q.

The purpose of the present work is the extension of the previous paper to include retardation. Thus we deal with the coupling between photons and plasmons to form 2D plasmon–polaritons in a thin semi-conductor film sandwiched between bulk media.

Our theory will be based on a common procedure in thin-film optics (Agranovich 1975, 1982) according to which the film is regarded as a transition layer between bulk media. The derivation of the boundary conditions for the fields outside the layer can be carried out directly within the framework of the Maxwell equations. The presence of the film will be taken into account only by these boundary conditions, which are more general than ordinary conditions, because they include the polarisability of the layer in the direction perpendicular to the interfaces. The theory provides both quantitative and qualitative links between the charge sheet (Nakayama 1974) and bulk slab models (Fuchs and Kliewer 1966, Mills and Maradudin 1973) of 2D plasmon–polaritons.

The outline of this paper is as follows. In § 2, we describe the macroscopic theory of transition layers between bulk dielectric media including effective boundary conditions, which take into account the dependence of the dielectric properties on the parameter qd. Section 3 consists of the application of the effective boundary conditions for derivation of the 2D plasmon–polariton dispersion relations. Both transverse magnetic (p) and transverse electric (s) polarisations are discussed. In § 4, we give alternative derivations of the dispersion equations by means of the macroscopic bulk slab models. Section 5 contains our comments and conclusions. The detailed derivation of the effective boundary conditions is given in the Appendix.

2. Effective boundary conditions for fields in the presence of a macroscopic homogeneous and isotropic transition layer

Let the z axis be normal to the semiconductor layer interfaces. Assume that the layer occupying region 3 ($0 \leqslant z \leqslant d$) has a background dielectric constant ε and contains free electrons distributed with an average surface concentration N. The thickness d is much larger than the lattice constant a, but much smaller than the light wavelength λ. The substrate half-space layer 1 ($z < 0$) is filled with a medium having a dielectric constant ε_1, and the half-space 2 ($z > d$) with a medium having a dielectric constant ε_2.

Electron motion parallel to the interface is considered free in the effective-mass approximation and the effective-mass tensor is taken to be diagonal with $m_x = m_y = m_t$, $m_z = m_l$. Motion perpendicular to the interface is quantised and, in the familiar model of a square well with infinite walls, the single-particle spectrum has the form

$$E_s = (\hbar^2\pi^2/2m_l d^2)s^2 \tag{2.1}$$

with wavefunctions

$$\psi_s(z) = (2/d)^{1/2}\sin(\pi sz/d) \tag{2.2}$$

where $s = 1, 2, \ldots$ is the sub-band number.

The wavefunction of an electron is the product of a planewave with vector $\boldsymbol{q}$ parallel to the interface and $\psi_s(z)$; so the total single-particle energy is

$$E_{sq} = E_s + \hbar^2 q^2/2m_t. \tag{2.3}$$

We assume that the electron plasma is weakly non-ideal, i.e. the effect of the Coulomb interaction is relatively small. For this to hold the electron gas must be sufficiently dense, i.e. $K_0 \gg a_0^{-1}$, where K_0 is the 2D Fermi momentum and a_0 is the effective Bohr radius (Chaplic 1971). For very low temperatures with only one sub-band filled, $K_0^2 = 2\pi N$ and the condition of weak non-ideality can be written $Na_0^2 \gg 1$, i.e. the number of electrons in the area of a Bohr orbit should be large. Our theory applies in this case only.

We take into account the polarisation of the film in the direction perpendicular to the boundaries. The electromagnetic response of the film is completely determined by the dielectric permittivities of the bounding media and this polarisation. In fact, the induced dielectric induction vector $\boldsymbol{D}$ is given by (Dahl and Sham 1977, Eguiluz and Maradudin 1978, Keldysh 1978)

$$D_i(q, \omega, z) = \varepsilon E_i(q, \omega, z) + 4\pi\delta_{ij}F_j(sz)\alpha_j(\omega, s) \times \int_0^l E_j(q, \omega, z')F_j^*(sz')\,\mathrm{d}z' \tag{2.4}$$

where $\boldsymbol{E}$ is the electric field and $i, j = x, y, z$. The summation convention applies.

We restrict attention to the case of substantially 2D plasmons and also ignore transitions between sub-bands, to which we hope to return in a later work.

For this case and in the long-wave approximation ($qd \ll 1$), α_j and F_j have the form (Dahl and Sham 1977)

$$\alpha = \alpha_x(\omega, 1) = \alpha_y(\omega, 1) = -\Omega_p^{2l}/\omega^2, \tag{2.5}$$

$$\alpha_z(\omega, 1) = 0, \tag{2.6}$$

$$F_{x,y}(1z) = d^{1/2}\psi_1(z)\psi_1(z) \tag{2.7}$$

$$F_z(1z) = 0 \tag{2.8}$$

$$\Omega_p^2 = 4\pi Ne^2/m_t. \tag{2.9}$$

Solutions of the Maxwell equations localised near the film are sought in the form

$$E_i(\boldsymbol{r}) = E_i\exp(\mathrm{i}qx - K_n|z|) \tag{2.10}$$

$$H_i(\boldsymbol{r}) = H_i\exp(\mathrm{i}qx - K_n|z|) \tag{2.11}$$

where $E_i \exp(-K_n|z|)$ and $H_i \exp(-K_n|z|)$ are the amplitudes of the electric and magnetic fields in the half-space n ($n = 1, 2$) and the x axis is taken as the direction of propagation. We consider non-radiative waves, for which

$$K_n^2 = [q^2 - (\omega^2/c^2)\varepsilon_n] > 0. \tag{2.12}$$

Following the usual method in the optics of thin films (Agranovich 1975) we use general effective boundary conditions derived in the Appendix and discussed by Agranovich (1982) and Keldysh (1978). From the Appendix these boundary conditions in the long-wave limit ($qd \ll 1$) are shown to be of the form

$$H_x^{(2)}(d) - H_x^{(1)}(0) = iqd[H_z^{(3)}(0) + H_z^{(3)}(d)]/2 - (i\omega/c)d(\varepsilon + 4\pi\alpha)[E_y^{(3)}(0) + E_y^{(3)}(d)]/2 \tag{2.13}$$

$$H_y^{(2)}(d) - H_y^{(1)}(0) = (i\omega/c)(\varepsilon + 4\pi\alpha)[E_x^{(1)}(0) + E_x^{(2)}(d)]/2 \tag{2.14}$$

$$E_x^{(2)}(d) - E_x^{(1)}(0) = iqd[E_x^{(2)}(d) + E_x^{(1)}(0)]/2 + (i\omega/c)d[H_y^{(2)}(d) + H_y^{(1)}(0)]/2 \tag{2.15}$$

$$E_y^{(2)}(d) - E_y^{(1)}(0) = (i\omega/c)d[H_y^{(1)}(0) + H_x^{(2)}(d)]/2 \tag{2.16}$$

where $E_i^{(1,2)}$ and $H_i^{(1,2)}$ are the plane boundary values of the electric and magnetic fields in the half-space 1 and 2. Here the presence of the film appears within the small quantities of the first order $qd \ll 1$.

3. 2D plasmon–polaritons in thin films

In this section, we derive the 2D plasmon–polariton dispersion relations in a semiconductor quantum film by means of (2.13)–(2.16). They separate into two disconnected sets, which are transverse magnetic (TM) (involving $\boldsymbol{H} \parallel \boldsymbol{q} \times \boldsymbol{n}$) and transverse electric (TE) (involving $\boldsymbol{E} \parallel \boldsymbol{q} \times \boldsymbol{n}$).

The derivation from (2.13) to (2.16) rests ultimately on (2.2) and the model of infinite walls. However, the results to be obtained are in fact more general, as can be seen from the derivation on the basis of entirely macroscopic considerations in § 4.

3.1. TM modes (p-polarisation)

In this case the electromagnetic-field boundary amplitudes from (2.10) and (2.11) are

$$H_y^{(1)}(0) = -(i\omega\varepsilon_1/cK_1)E_x^{(1)}(0) \tag{3.1}$$

$$H_y^{(2)}(d) = (i\omega\varepsilon_2/cK_2)E_x^{(2)}(d) \tag{3.2}$$

$$E_y = H_x = H_z = 0 \tag{3.3}$$

where c is the velocity of light.

Substitution of (3.1) and (3.2) in (2.13) and (2.14), with use of (2.5) for α, gives the dispersion relation

$$\varepsilon_1/K_1 + \varepsilon_2/K_2 = [(\Omega_p/\omega)^2 - \varepsilon]d. \tag{3.4}$$

Before discussing the dispersion law $\omega = \omega(K)$ from (3.4), it is useful to note some relevant features.

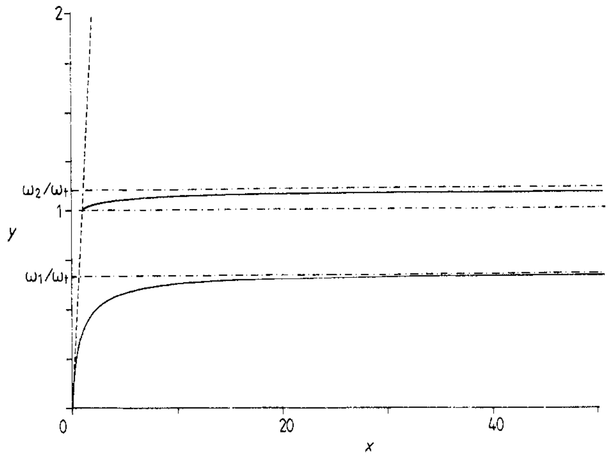

Figure 1. The dispersion relation of TM 2D plasmon–polaritons for GaAs with $d = 150$ Å and $N = 0.5 \times 10^{12}\,\mathrm{cm}^{-2}$ embedded in media with $\varepsilon_1 = 13$: ——, dispersion curves; – – –, light line; —·—, frequency values ω_1, ω_t and ω_2. Both the frequency and the wavenumber are plotted in reduced units $y = \omega/\omega_t$ and $x = q/q_t$, where $\omega_t = cq_t/\varepsilon_1^{1/2}$. The values of ω_t, ω_l and q_t are illustrated in table 1.

In the unretarded limit ($c \to \infty$), (3.4) reduces to

$$\omega = \{(4\pi Ne^2/m_t\varepsilon)q/[qd + (\varepsilon_1 + \varepsilon_2)/\varepsilon]\}^{1/2} \tag{3.5}$$

as derived previously by means of RPA method (Aharonian *et al* 1988). For $qd \ll 1$ and $\varepsilon \sim \varepsilon_{1,2}$, (3.5) becomes

$$\omega = \{[4\pi Ne^2/m_t(\varepsilon_1 + \varepsilon_2)]q\}^{1/2} \tag{3.6}$$

which is the result of Stern (1967).

For $\varepsilon \gg \varepsilon_{1,2}$ and $qd \ll 1$ the dispersion relation takes the form

$$\omega = \{[4\pi Ne^2/m_t(\varepsilon_1 + \varepsilon_2)]q\}^{1/2} \qquad qd \ll (\varepsilon_1 + \varepsilon_2)/\varepsilon \ll 1 \tag{3.7}$$

and

$$\omega = \{4\pi Ne^2/m_t\varepsilon d\}^{1/2} \qquad (\varepsilon_1 + \varepsilon_2)/\varepsilon \ll qd \ll 1. \tag{3.8}$$

Thus the EICH on the 2D plasmon dispersion spectra is significant for $\varepsilon_{1,2} \ll \varepsilon$ only.

It can be seen that for certain values of ω and d the right-hand side of (3.4) may become negative, in which case the non-radiative TM mode dispersion spectrum possesses some gaps. On the contrary as $d \to 0$ in (3.4), we recover the 2D charge sheet non-radiative dispersion relationships (Nakayama 1974), which have solutions for any ω.

The position of any gaps is found from the condition

$$(\Omega_p/\omega)^2 - \varepsilon = 0. \tag{3.9}$$

By way of example, let the medium $0 \leqslant z \leqslant d$ be a polar semiconductor with ε of the form

$$\varepsilon = \varepsilon_\infty(\omega^2 - \omega_l^2)/(\omega^2 - \omega_t^2) \tag{3.10}$$

where ω_l and ω_t are the longitudinal and transverse optical phonon frequencies and ε_∞ is the semiconductor high-frequency dielectric constant.

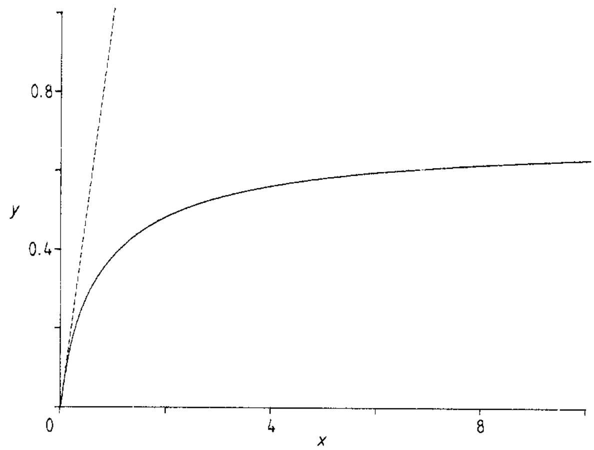

Figure 2. The low-frequency dispersion curve of TM 2D plasmon–polaritions for a GaAs film on an expanded scale.

Table 1. Characteristic parameters for GaAs with $d = 150$ Å and $N = 0.5 \times 10^{12}$ cm^{-2}.

	m^*/m_e	ε_s	ε_∞	$\omega_l\ (10^{-13}\ \mathrm{s}^{-1})$	$\omega_t\ (10^{13}\ \mathrm{s}^{-1})$	$\Omega_p\ (10^{13}\ \mathrm{s}^{-1})$	ε_1	$q_T\ (10^3\ \mathrm{cm}^{-1})$
GaAs	0.063	12.9	10.9	5.52	5.23	13.03	13	3.02

By solving (3.9) with (3.10), we find the roots ω_1 and ω_2 of (3.9) which are

$$\omega_{1,2}^2 = \omega_l^2/2 + \frac{\Omega_p^2}{2\varepsilon_\infty} \pm [(\omega_l^2/2 + \Omega_p^2/2\varepsilon_\infty)^2 - \Omega_p^2\omega_t^2/\varepsilon_\infty]^{1/2}. \tag{3.11}$$

We now investigate the dispersion law $\omega = \omega(K)$ in the symmetric geometry $\varepsilon_1 = \varepsilon_2$, for which comparatively convenient analytic expressions can be found. In particular, (3.4) yields the solution

$$q = \varepsilon_1^{1/2}(\omega/c)\{1 + A\omega^2(\omega^2 - \omega_t^2)^2/[B(\omega^2 - \omega_t^2) - \omega^2(\omega^2 - \omega_l^2)]^2\}^{1/2} \tag{3.12}$$

where

$$A = 4\varepsilon_1 c^2/\varepsilon_\infty^2 d^2 \qquad B = \Omega_p^2/\varepsilon_\infty.$$

We illustrate the properties of the dispersion law (3.12) by means of a dispersion graph. As our example we choose a GaAs film. The results are shown in figures 1 and 2 with reduced units $y = \omega/\omega_t$ and $x = q/q_t$ where $q_t = (\omega_t/c)\varepsilon_1^{1/2}$. Values of the parameters used are given in table 1.

From figure 1, we can see that the dispersion spectra of TM plasmon–polaritons in thin films possess gaps and the frequency intervals $0 \leqslant \omega < \omega_1$ ($0 \leqslant \omega/\omega_t < \omega_1/\omega_t$ in the graph) and $\omega_t \leqslant \omega < \omega_2$ ($1 \leqslant \omega/\omega_t < \omega_2/\omega_t$ in the graph) are allowed regions for the non-radiative plasmon–polariton TM waves. It follows from (3.11) that, in the formal limit $d \to \infty$, we have $\omega_1 \to 0$ and $\omega_2 \to \omega_l$. So in that case the allowed frequency interval is $\omega_t \leqslant \omega \leqslant \omega_l$ only, which corresponds to the surface-localised modes in the bulk slab model (Ushioda and Loudon 1982, Cottam and Tilley 1989). In the opposite case when

Table 2. The numerical boundary values of the allowed frequency regions $y_1 = \omega_1/\omega_t$ and $y_2 = \omega_2/\omega_t$ and also the width $\Delta y_1 = 1 - y_1$ of the gap between the first and second allowed regions and the width $\Delta y_2 = y_2 - 1$ of the second allowed region.

	y_1	y_2	Δy_1	Δy_2
GaAs	0.685	1.102	0.315	0.102

$d \to 0$ in (3.4) we recover the 2D charge sheet dispersion curve (Nakayama 1974), which does not possess any gaps in the frequency spectra. So equation (3.4) with the corresponding graph contains both the bulk and the charge sheet dispersion patterns in the long-wave limit ($qd \ll 1$). This means that our model provides the link between these models.

The value of the gap $\Delta\omega_1 = \omega_t - \omega_1$, as follows from (3.11), depends on the quantities Ω_p, ω_t, ω_l and ε_∞. For given m^*, $\Delta\omega_1$ is large when ε_∞ is large and increases with decreasing N. Values are given in table 2.

We now discuss further some characteristic features of the dispersion law (3.12) with reference to figure 2.

(i) When $\omega \ll \omega_t$, Ω_p as follows from (3.12), the spectral dependence is

$$q = \varepsilon_1^{1/2}(\omega/c) \tag{3.13}$$

where the dispersion law depends on only the substrate dielectric constant. From (3.13) we conclude that for the same values of the wavenumber q with decreasing values of ε_1 the frequency values are increasing and the dispersion curve becomes closer to the frequency axis (the y axis in the graphs). Therefore the radiative area, which lies to the left-hand side of the light line (3.13), is decreasing.

(ii) In the particular case $\Omega_p^2 \gg \varepsilon\omega^2$ (ε is defined in (3.10)) the spectral law $\omega = \omega(q)$ is given by

$$q = (2\varepsilon_1/\Omega_p^2 d)\omega^2. \tag{3.14}$$

Numerical estimates show that the law (3.14) applies in GaAs for wavevectors of about $10^5\ \mathrm{cm}^{-1}$ (it corresponds to values $x \geq 20$ in figures 1 and 2). It is further seen from figure 2 that the dispersion curve departs from the light line (3.13) for relatively low frequencies in accordance with (3.12).

(iii) In the allowed frequency interval $\omega_t \leq \omega < \omega_2$ ($1 \leq \omega/\omega_t < \omega_2/\omega_t$ in the graph) the spectral curve departs from the light line (3.13) quite abruptly as characterised by law (3.12).

As we can see from figure 1 the dispersion curve diverges for the values ω_1/ω_t and ω_2/ω_t, which are defined by (3.11); the first dispersion curve in the region $0 \leq \omega/\omega_t < \omega_1/\omega_t$ represents the 2D plasmon.

3.2. TE *modes (s-polarisation)*

The TE electromagnetic-field boundary amplitudes are

$$H_x^{(1)}(0) = (\mathrm{i}cK_1/\omega)E_y^{(1)}(0) \tag{3.15}$$

$$H_x^{(2)}(d) = -(\mathrm{i}cK_2/\omega)E_y^{(2)}(d) \tag{3.16}$$

$$H_z^{(1)}(0) = (cq/\omega)E_y^{(1)}(0) \tag{3.17}$$

$$H_z^{(2)}(d) = (cq/\omega)E_y^{(2)}(d) \tag{3.18}$$

$$E_x = E_z = H_y = 0. \tag{3.19}$$

After substituting in the boundary conditions, we find the dispersion relationship for the TE plasmon–polarition modes in the form

$$K_2 + K_1 = [\varepsilon(\omega^2/c^2) - q^2 - \Omega_p^2/c^2)d. \tag{3.20}$$

As in the TM case we shall assume that $\varepsilon_1 = \varepsilon_2$. In this case, (3.20) has the following solution in the long-wave limit ($qd \ll 1$):

$$q = [\![\{4\varepsilon_1(\omega^2/c^2) - \varepsilon_\infty^2(\omega^4/c^4)d^2[(\omega^2 - \omega_l^2)/(\omega^2 - \omega_t^2) - \Omega_p^2/\omega^2\varepsilon_\infty]^2\} \times \{4 + 2\varepsilon_\infty(\omega^2/c^2)d^2[(\omega^2 - \omega_l^2)/(\omega^2 - \omega_t^2) - \Omega_p^2/\omega^2\varepsilon_\infty]\}^{-1}]\!]^{1/2} \tag{3.21}$$

for the frequencies ω, for which the right-hand side of (3.20) is not negative. We have made numerical calculations of dispersion graphs based on (3.21). In all cases, however, the dispersion curves are very close to the light line $q = \varepsilon_1^{1/2}\omega/c$, and they are therefore not shown. This simple behaviour arises because of the dominance of the first terms in both the numerator and the denominator of (3.21).

4. Alternative derivation of the 2D plasmon–polariton dispersion relations in quantised films

We now show how the 2D plasmon–polariton dispersion relations can be derived from a bulk slab model. As before, two plane interfaces are situated at $z = 0$ and $z = d$. The medium 1 ($z < 0$) has a dielectric constant ε_1 and medium 2 ($z > d$) a dielectric constant ε_2. The semiconductor slab occupies the region $0 \leqslant z \leqslant d$ and has a dielectric constant ε_3. As is well known, (Ushioda and Loudon 1982, Cottam and Tilley 1989) for a sufficiently large thickness, d-independent p-polarised surface polaritons propagate on each interface in the appropriate frequency range. As d decreases, the fields of the two surface polaritons propagating for a given wavevector $\boldsymbol{q}$ start to overlap, and the degeneracy in frequency between them is lifted. Thus we expect to see two modes, essentially a bonding and anti-bonding combination. The quantities K_1 and K_2 from (2.12) must always be real (in the absence of damping), and

$$K_3 = [q^2 - \varepsilon_3(\omega^2/c^2)]^{1/2} \tag{4.1}$$

is real for the surface-mode case just described. In addition, however, K_3 can be imaginary, in which case the z dependence of the fields in the medium $0 \leqslant z \leqslant d$ is oscillatory, so that the medium is behaving like a waveguide. Thus in p polarisation in a slab both surface and guided-wave polaritons may occur.

In s polarisation, on the contrary, only guided-wave polaritons are found.

4.1. TM *polaritons (p polarisation)*

The dispersion relation for this mode has the form (see, e.g. Cottam and Tilley 1989)

$$\exp(-2K_3d) = [r_3^2 + r_1r_2 + r_3(r_1 + r_2)]/[r_3^2 + r_1r_2 - r_3(r_1 + r_2)] \tag{4.2}$$

where

$$r_i = \varepsilon_i/K_i \qquad (i = 1, 2, 3).$$

Equation (4.2) applies to both surface polaritons and guided-wave modes depending on whether K_3 is real or imaginary.

After a few simple transformations, we can get from (4.2)

$$[\exp(-2K_3d) + 1]/[\exp(2K_3d) - 1] = (r_3^2 + r_1r_2)/r_3(r_1 + r_2). \tag{4.3}$$

Expanding (4.3) to lowest order in $K_3d \ll 1$ and solving for r_3, we find that

$$r_3 = -(1/K_3d)(r_1 + r_2). \tag{4.4}$$

If we substitute the expressions for r_i in (4.4), we obtain

$$\varepsilon_1/K_1 + \varepsilon_2/K_2 = -\varepsilon_3 d. \tag{4.5}$$

Now we assumne that ε_3 has a plasma-like part:

$$\varepsilon_3 = \varepsilon - \Omega_p^2/\omega^2 \tag{4.6}$$

where ε is the background dielectric constant and Ω_p is the plasma frequency (2.9):

$$\Omega_p^2 = 4\pi Ne^2/m_t d \tag{4.7}$$

since N is the areal density.

If we combine equations (4.5)–(4.7), we get the dispersion relation (3.4).

This derivation holds for both real and imaginary values of K_3 and in effect means that, in very thin films, guided and surface polariton slab modes coincide.

4.2. TE *polaritons (s polarisation)*

The derivation of the dispersion relation for TE polarised guided-wave modes is similar to that for TM modes. It emerges that the equations for p polarisation can be converted to those for s polarisation by the substitutions (Cottam and Tilley 1989)

$$K_2\varepsilon_3/K_3\varepsilon_2 \rightarrow K_3/K_2 \qquad K_1\varepsilon_3/K_3\varepsilon_1 \rightarrow K_3/K_1 \tag{4.8}$$

Note, however, that, since only guided modes occur, K_3 is pure imaginary.

Thus, substituting (4.8) into (4.5) in the lowest order $K_3d \ll 1$, we have

$$K_1 + K_2 = -K_3^2 d. \tag{4.9}$$

With the use of (4.1) and (4.6) this is seen to reduce exactly to (3.20).

5. Conclusions

The theory outlined above for the 2D plasmon–polariton modes in thin quantised but finite films provides the link between the charge sheet (Nakayama 1974) and bulk slab (Mills and Maradudin 1973, Fuchs and Kliewer 1966) models used previously. Here it has been shown that TM polarised 2D plasmon–polaritons in thin but finite films are the limit of either surface or guided modes, which are coincident in this case. The most striking feature is the existence of energy gaps in the dispersion spectra. The magnitude of these spectral gaps depends on the film thickness, carrier surface concentration and dielectric constants of the neighbouring media. For the TE polarised modes the coupling between the external electromagnetic field and carriers in the film is weak and the dispersion law in very close to the light line.

Acknowledgment

KHA thanks the British Council for support for a period of study leave at the University of Essex.

Appendix

We give here the derivation of the effective boundary conditions which were used in §§ 2 and 3. As before in § 2, we take the z axis normal to the interfaces, which are situated at $z = 0$ and $z = d$. The x axis is taken along the direction of wave propagation. The monochromatic fields $\boldsymbol{E}$, $\boldsymbol{D}$ and $\boldsymbol{H}$ vary in space according to the law

$$A(\boldsymbol{r}, t) = A(z)\exp[\mathrm{i}(qx - \omega t)]. \tag{A1}$$

In order to obtain the boundary conditions we integrate the Maxwell equations with respect to z between the limits $z = 0$ and $z = d$. For the components of the magnetic field strength $\boldsymbol{H}$, we make use of the equation

$$\nabla \times \boldsymbol{H} = -(\mathrm{i}\omega/c)\boldsymbol{D}. \tag{A2}$$

For the xth component of the fields of the type (A1), we have from (A2)

$$\partial H_x(z)/\partial z - \mathrm{i}qH_z(z) = -(\mathrm{i}\omega/c)D_y \tag{A3}$$

which, after integrating together with (2.4) with respect to z between the limits $z = 0$ and $z = d$, becomes

$$H_x^{(3)}(d) - H_x^{(1)}(0) = \mathrm{i}q\int_0^d H_z^{(2)}(z)\,\mathrm{d}z - \frac{\mathrm{i}\omega}{c}\,\varepsilon\int_0^d E_y^{(3)}(z)\,\mathrm{d}z$$
$$- 4\pi\alpha_y(\omega, 1)\int_0^d F_y(1z)\,\mathrm{d}z\int_0^d E_y^{(3)}(z')F_y(1z')\,\mathrm{d}z'. \tag{A4}$$

The quantities $H_i^{(n)}(d)$ (as well as $E_i^{(n)}(d)$) in (A4) can be formally represented as the expansion

$$H_i^{(n)}(d) = H_i^{(n)}(0) + \mathrm{d}[\partial H_i^{(n)}(0)/\partial z]|_{z=0} + \ldots. \tag{A5}$$

Now we follow the usual method in the theory of the transition layer and take into consideration only the leading terms in the small quantities qd and $(\omega/x)d$.

Since the boundary conditions

$$H_x^{(1)}(0) = H_x^{(3)}(0) \qquad H_x^{(2)}(d) = H_x^{(3)}(d) \tag{A6}$$

apply, expansion of (A4) to lowest order gives

$$H_x^{(2)}(d) - H_x^{(1)}(0) = \mathrm{i}qH_z^{(3)}(0)d - (\mathrm{i}\omega/c)d(\varepsilon + 4\pi\alpha)E_y^{(3)}(0). \tag{A7}$$

In fact, (A7) can be used in the more convenient form

$$H_x^{(2)}(d) - H_x^{(1)}(0) = \mathrm{i}qd[H_z^{(3)}(0) + H_z^{(3)}(d)]/2$$
$$- (\mathrm{i}\omega/c)d(\varepsilon + 4\pi\alpha)[E_y^{(3)}(0) + E_y^{(3)}(d)]/2 \tag{A8}$$

which is identical with the lowest order.

In the same way, we derive for the H_y component the boundary condition

$$H_y^{(2)}(d) - H_y^{(1)}(0) = (\mathrm{i}\omega/\mathrm{c})(\varepsilon + 4\pi\alpha_y)[E_x^{(1)}(0) + E_x^{(2)}(d)]/2. \tag{A9}$$

For the components of electric field $\boldsymbol{E}$, similar manipulations produce from the equation

$$\nabla \times \boldsymbol{E} = (\mathrm{i}\omega/c)\boldsymbol{H} \tag{A10}$$

the boundary conditions

$$E_x^{(2)}(d) - E_x^{(1)}(0) = \mathrm{i}qd[E_x^{(2)}(d) + E_x^{(1)}(0)]/2 + (\mathrm{i}\omega/c)d[H_y^{(2)}(d) + H_y^{(1)}(0)]/2 \tag{A11}$$

$$E_y^{(2)}(d) - E_y^{(1)}(0) = -(\mathrm{i}\omega/c)d[H_x^{(1)}(0) + H_x^{(2)}(d)]/2. \tag{A12}$$

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